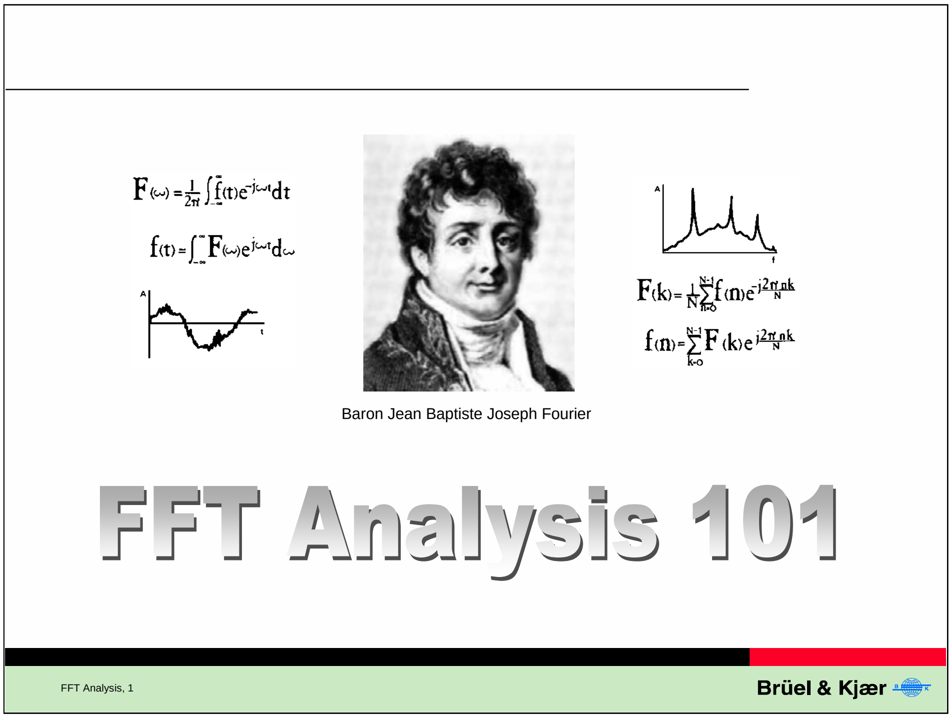


Abstract

This lecture will introduce the basic principles of FFT analysis. Since this is an introductory lecture only, some more advanced areas are not covered in depth. For more in depth coverage please see the references page at the end or attend a full day Bruel & Kjaer seminar on DSP analysis (for dates and locations go to http://www.bkhome.com/seminars).

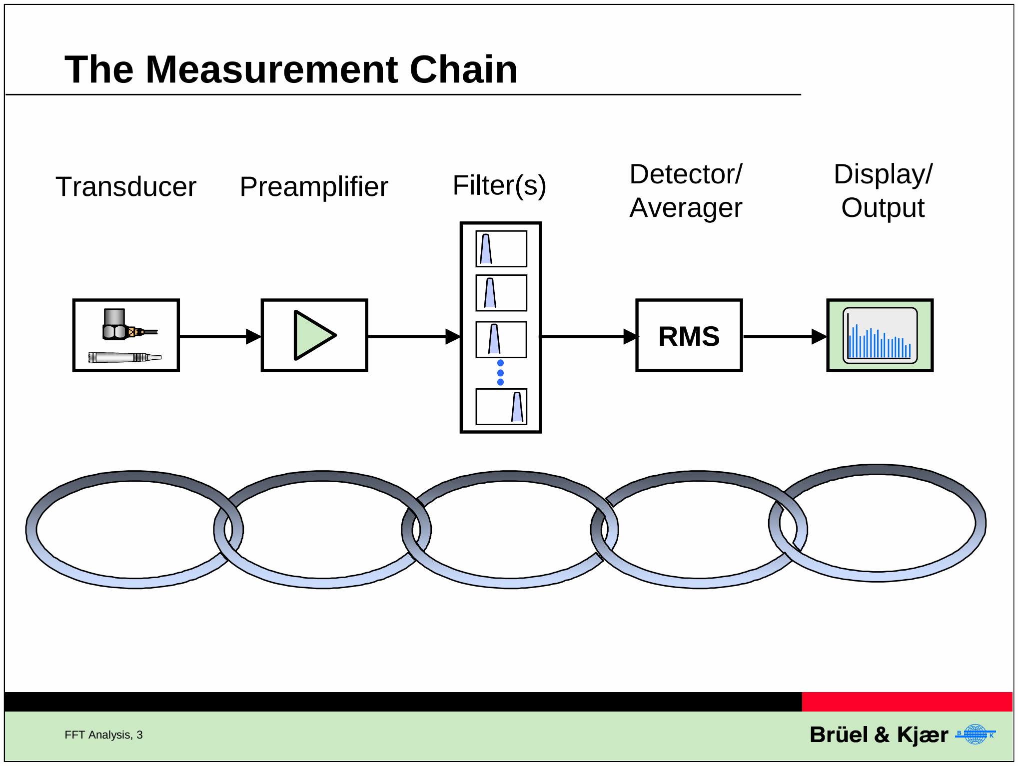
LECTURE NOTE

BA 7683-11

Brüel & Kjær

FFT Analysis 101

- Introduction
- Practical Set Up of FFT Analysers
- Pitfalls of an FFT Analyser
- Real-time Analysis
- Time Weighting
- Overlap Analysis
- Signal Types and Spectrum Units
- FFT Summary



The Measurement Chain

Frequency analysis is performed using some type of frequency analyzer which can have several different forms, from a simple Sound Level Meter with an octave filter bank, to large computer based systems which can perform a variety of different forms of narrowband analysis and complex postprocessing. However, in principle they all contain these basic building blocks listed below:

1. The transducer converting the original signal into an electric signal.

2. The preamplifier amplifying and conditioning the electric signal.

3. The filter(s) separating the different frequencies in the time signal.

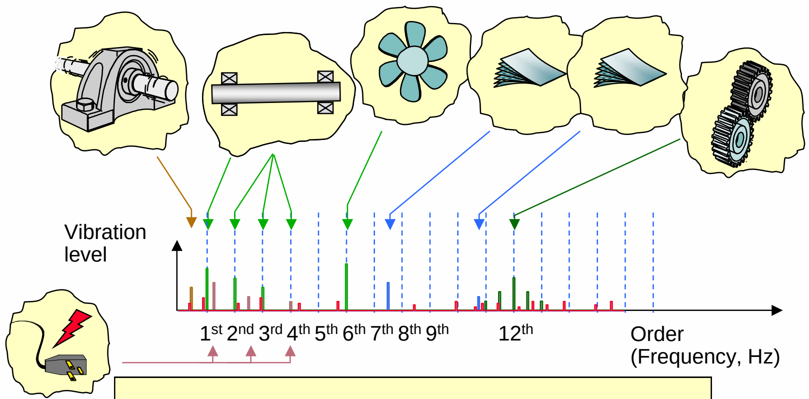
4. The detector which makes a power/energy related average of the filter output.

5. The output which can be viewed on some form of display or computer monitor, or can be transferred to an output device, such as a printer or storage device.

Transducers and preamplifiers will not be described in this lecture and before we begin discussing the other blocks in the measurement chain, we will look at the characteristics of different signals.

Sources of Machine Vibration

The moving parts of machines create vibration at different frequencies.



Many different vibrations compose a single frequency spectrum

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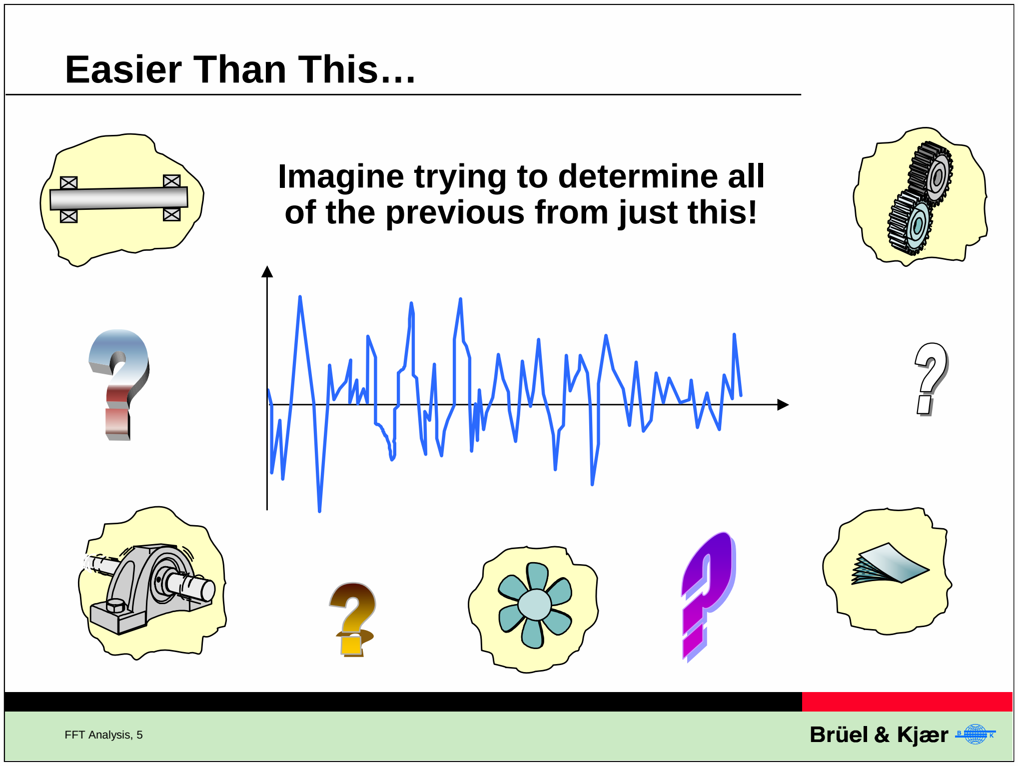
Sources of Machine Vibration

In this slide we can see a typical frequency spectra of a rotating machine. There are many different sources that contribute to the frequency spectra. Each source produces distinct frequencies and amplitudes that are easily seen when viewed this way…



Easier Than This!

…but when we view the raw time signal it is very difficult to correlate frequency and amplitude information back to the sources! This is the whole point of frequency analysis, DATA REDUCTION! We take a sophisticated time signal, which has a lot of information all mixed together, and reduce it down to individual frequency and amplitude information.

The Fourier Transform

$$G(f) = \int_{-\infty}^{+\infty} g(t)\, e^{-j2\pi f t} dt$$

$$g(t) = \int_{-\infty}^{+\infty} G(f)\, e^{j2\pi f t} df$$

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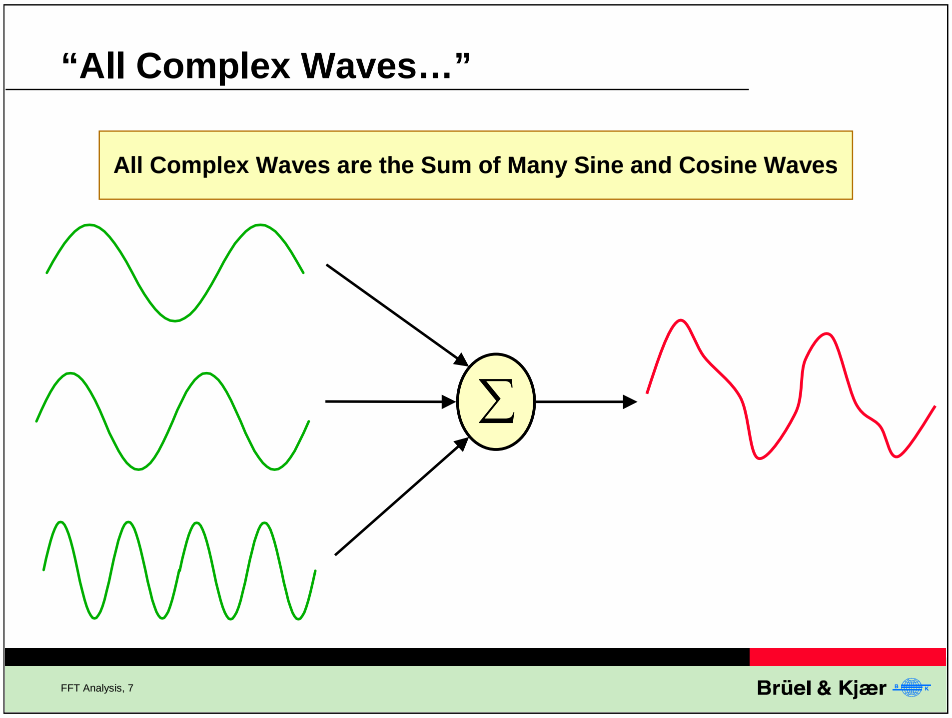
The Fourier Transform

We can use the Fourier transform to perform a frequency analysis. The time signal is multiplied by some complex number and then integrated over all time. This complex number is actually a frequency shift from any frequency of interest down to DC. Then the average value of this DC is extracted.

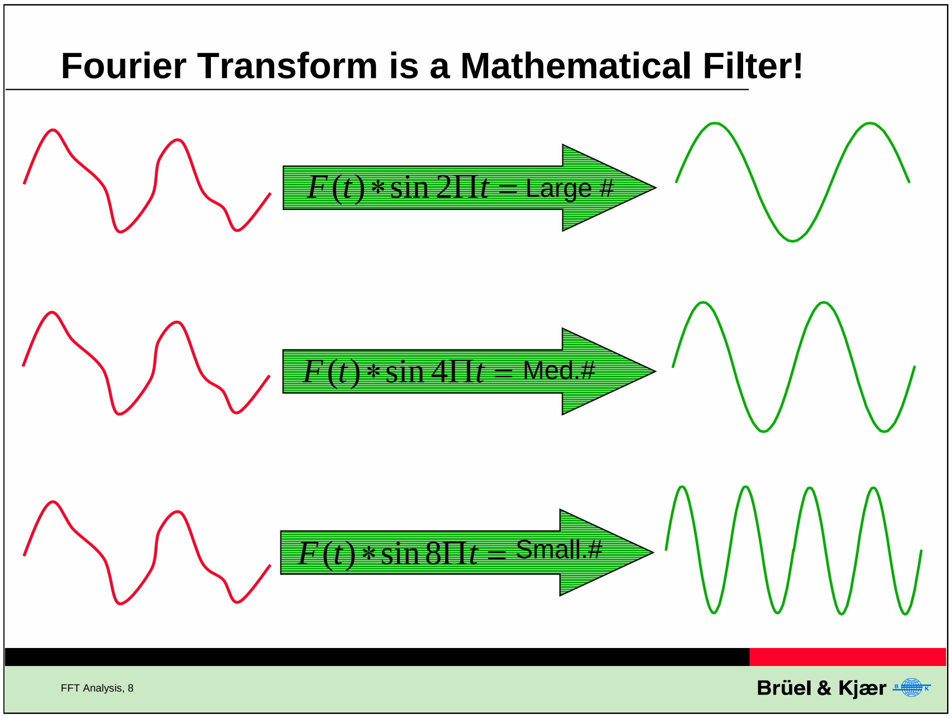
One problem is already seen — we have to know the signal from minus to plus infinity. As shown in the next slide, in practical situations we have to time limit the system. Otherwise, we would never be able to finish a single frequency analysis in our lifetime.

The Fourier transform is a reversible transform. Using the inverse Fourier transform brings the Fourier spectrum back to the time domain again. Notice the symmetry between the forward and the inverse Fourier transform.



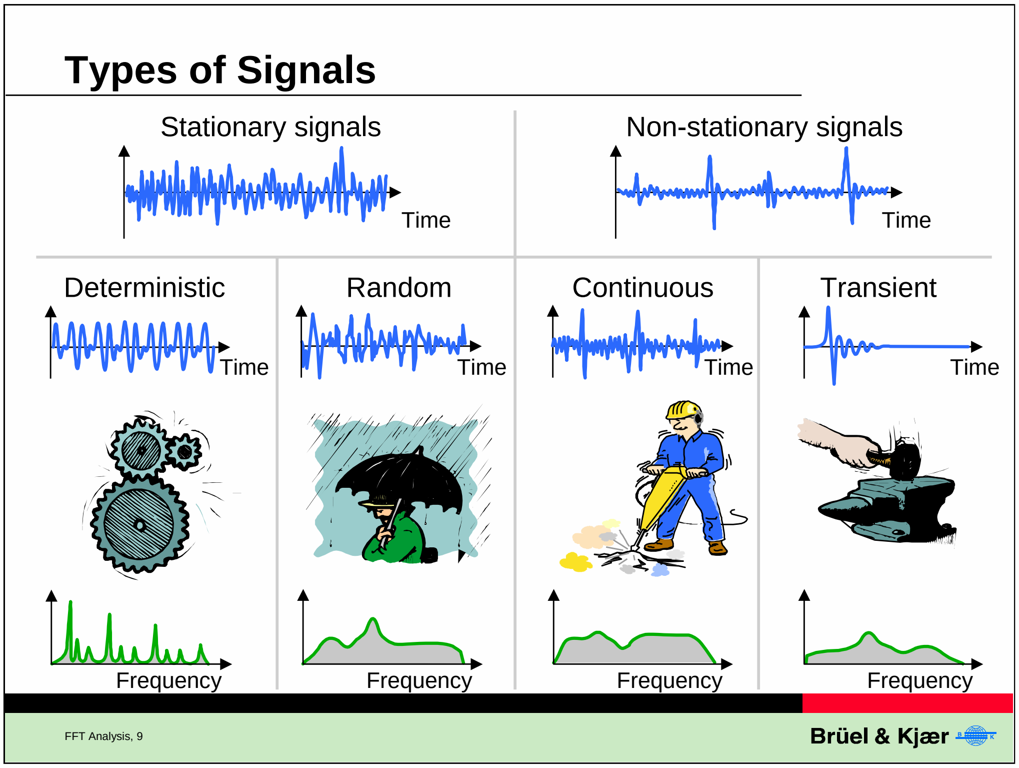
All Complex Waves…

The basis of the FFT is the fact that all complex time signals (as seen in red) are a sum of many different sine and cosine waves at varying frequencies and amplitudes. When those sine and cosine waves are summed together they produce the sophisticated time signal.



Fourier Transform is a Mathematical Filter!

The FFT analyzer uses the 'sum' principle from the previous slide to its advantage. By passing the sophisticated time signal through some relatively simple math we can filter out a specific sine wave. Once that sine wave is filtered, we measure the amplitude and plot that in an frequency spectra. KEEP IN MIND that FFT analysis is much more sophisticated that this simple example, but this is the basic idea!



Signals

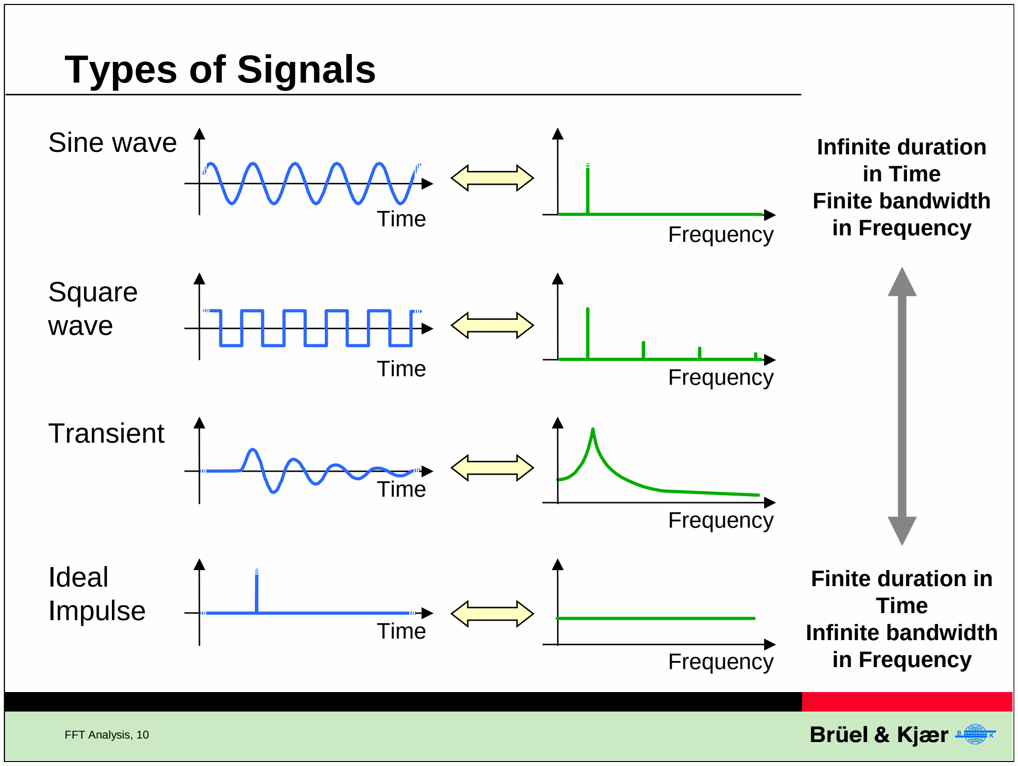
When looking at real life signals we basically have to make a distinction between Stationary and Non-stationary signals. Stationary signals can again be divided into Deterministic and Random signals, and Non-stationary signals into Continuous and Transient signals.

Deterministic signals are made up entirely of sinusoidal components at discrete frequencies. When the spectral lines show a harmonic relationship, the signal is described as being periodic. An example of a periodic signal is vibration from a rotating shaft. Where no harmonic relationship exists, the signal is described as being quasi-periodic.

A random signal is a continuous stationary signal whose properties can only be described using statistical parameters. Examples of random signals are the effects of cavitation and turbulence. Random signals have a frequency spectrum which is continuously distributed with frequency.

The continuous Non-stationary signal has some similarities with both transient and stationary signals. During analysis continuous non-stationary signals should normally be treated as random signals or separated into their individual transients and treated as transients.

A transient signal is a signal which only exists for a short period of time. Examples of transient signals are combustion in a reciprocating machine, or the opening or closing of a valve. Both random and transient signals produce continuous spectra.



Types of Signals

In order to make a proper frequency analysis of signals it is necessary to know the characteristics of the different types of signals.

As illustrated here, all signals have characteristics which lie between the two extremes: a sine wave and an ideal impulse. An ideal sine wave which in theory has infinite duration in time (from $-\infty$ to $+\infty$) will in the frequency domain be represented by a single infinitely narrow value. In contrast to this, an ideal pulse which is infinitely narrow in time will in the frequency domain have a value at all frequencies.

What is broad in one domain is narrow in the other

Between these two extremes we have all other signals. Two examples are shown here:

• a square wave which is periodic in time will have its energy concentrated at specific frequencies

and

• a transient response of a Single Degree of Freedom system which, because of its periodic nature, has most of its energy concentrated at one frequency and, because it is limited in time, has energy over a wide frequency range.

Think of these simple principles as “rules of thumb” when looking at signals in the following chapters.

FFT Analysis 101

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What is the Fast Fourier Transform?

- An algorithm for increasing the speed of the computer calculation of the Discrete Fourier transform.
  - Reduces the number of multiplications from N^2 to $(N/2)\log_2 N$
  - Computation speed increased by a factor of 372 for an 800 line FFT
- Block analysis of time data samples to provide equivalent frequency domain description
- Analysis with constant bandwidth filters
- "Rediscovered" in 1962 by Bell Lab scientists Cooley and Tukey

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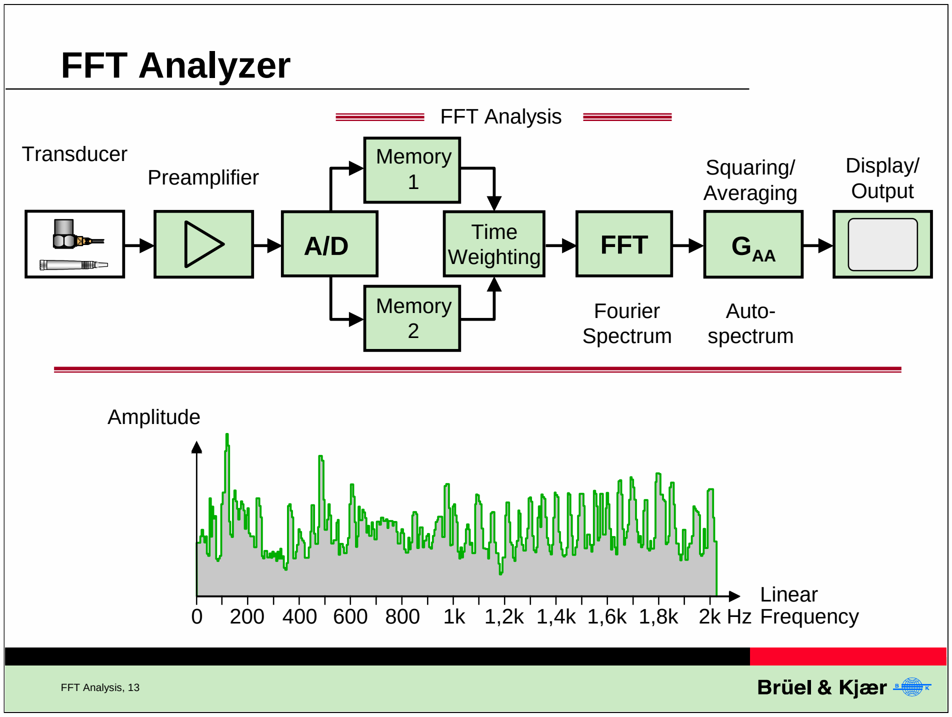
Features of FFT Analysis

An FFT (Fast Fourier Transform) analyser is based on the principle of a fast and efficient calculation of the Discrete Fourier Transform. As discussed earlier is the Discrete Fourier Transform discrete both in the time and in the frequency domain, thus enabling a digital processor to make a direct calculation. Most efficient calculations are done if the transform size is a power of 2. Most often transform sizes are $N = 2^{10} = 1024$ and $N = 2^{11} = 2048$ producing frequency spectra of respectively 400 and 800 lines.

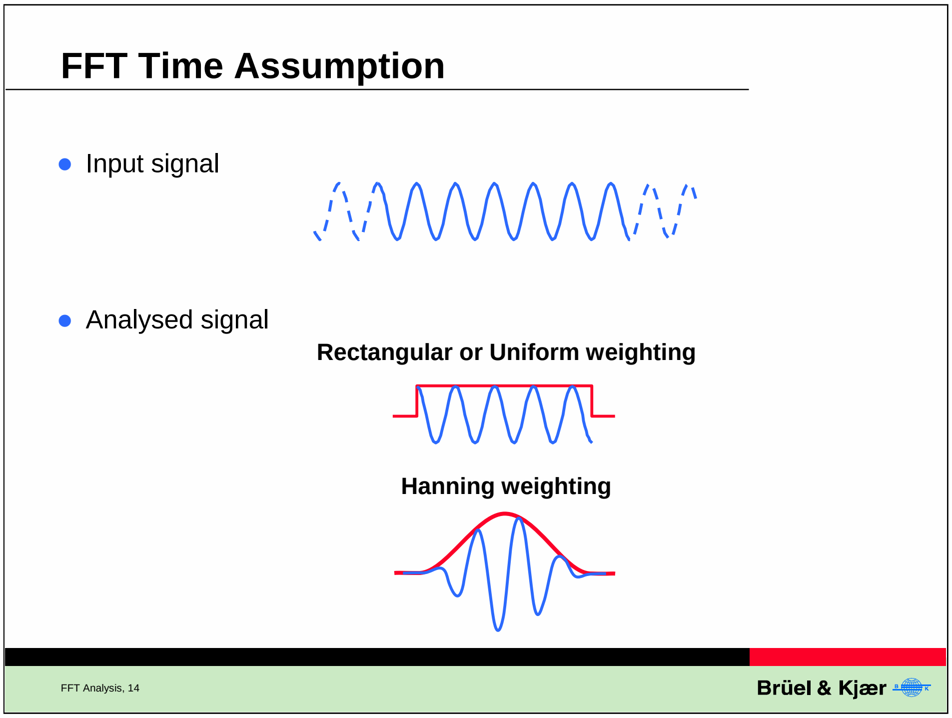
The line spectrum resulting from an FFT analysis is equidistant, so the time signal is analysed in constant bandwidths.

In an FFT analyser the time signals are analysed in blocks. A block of time data is recorded in a memory and a Fast Fourier Transform of the whole block is performed.



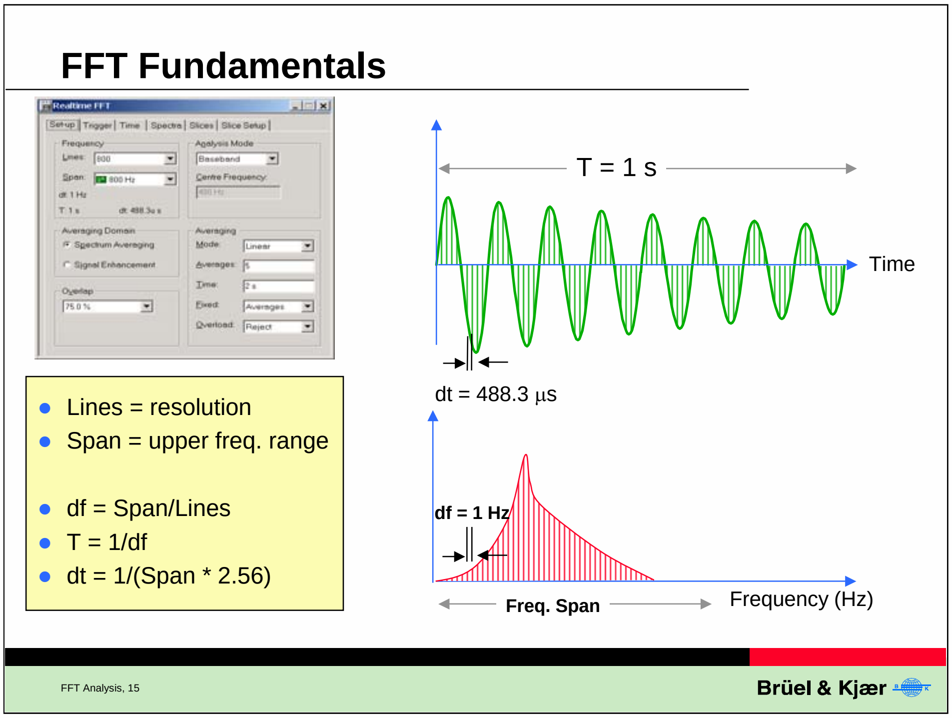
Block Diagram of an FFT Analyser

A simplified block diagram of an FFT analyzer is shown here. The input signal is amplified and filtered by a low pass filter in order to avoid aliasing. After sampling and conversion to digital form, the signal is fed to the memory. A dual memory is used in order to obtain real-time processing. While FFT processing is performed on the content of one memory, new data is recorded in the other memory (real-time recording is discussed shortly). After multiplication with a suitable weighting function the Fast Fourier Transformation, FFT is performed and the Fourier Spectrum is multiplied by its complex conjugate (complex squaring) to give the Auto-spectrum before it is finally displayed.



FFT Time Assumption

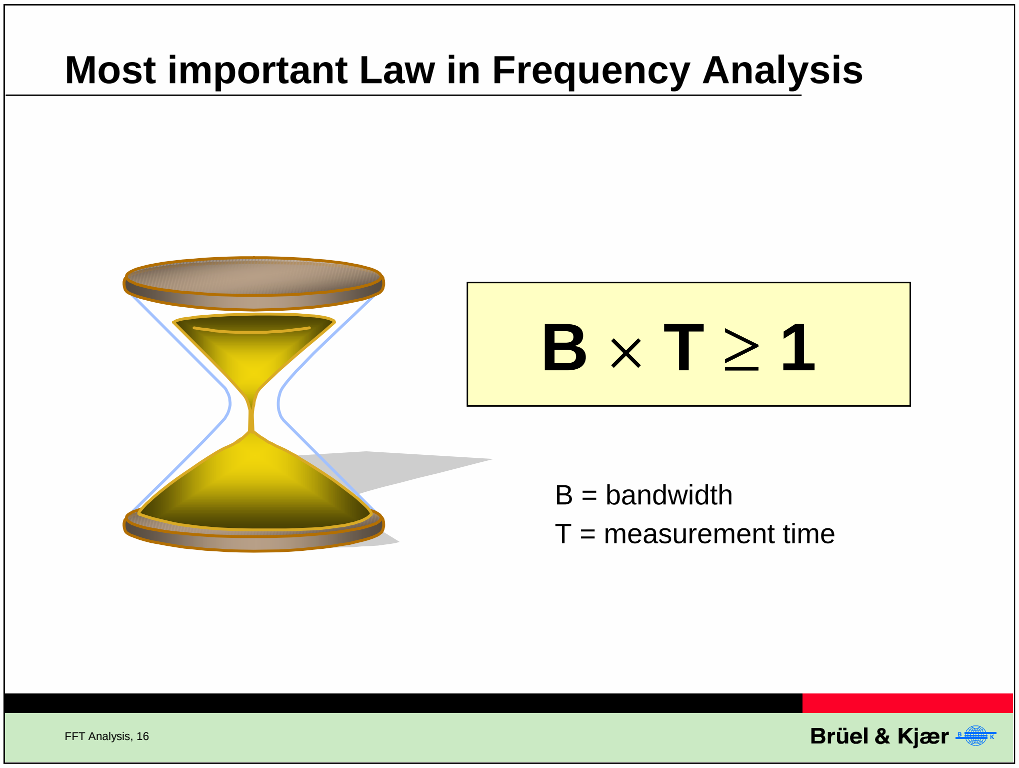
Using a Fourier analyzer we time limit the signal before the analysis takes place. One possibility is to cut the signal into pieces using no weighting at all, some times also called Rectangular or Uniform weighting. Unfortunately this creates some discontinuities in the beginning and the end of the record, which creates some undesirable effects in the corresponding frequency spectra. These effects are minimized (not completely eliminated) using a smoother weighting function which attenuate the signal amplitude down to zero at the beginning and the end of the record. The most widely used weighting is Hanning, which is one period of a cosine.



FFT Fundamentals

Here we can see the basic settings of an FFT Analyzer. We always have to specify the Lines and Resolution when setting up an FFT. Once these parameters are specified, the df (resolution in the frequency spectra), T (time block used to gather data to calculate one FFT spectra), and dt (sampling time required), are automatically calculated behind the scenes.

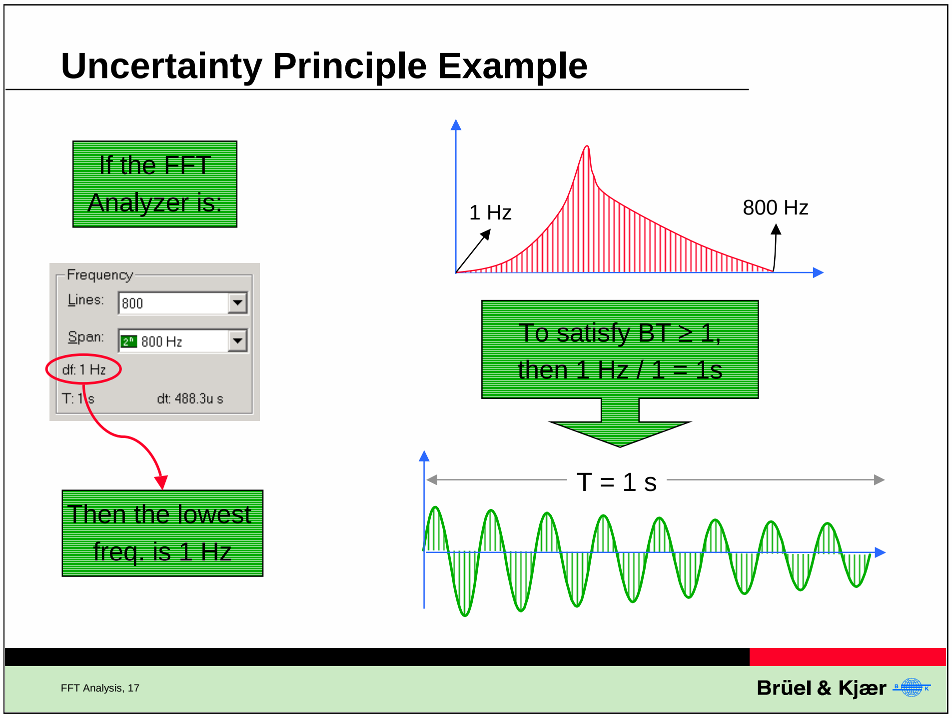
Notice the inverse relationship between df and T. This is a fundamental thing to remember: when you use more frequency resolution you require longer time blocks and vice versa. This will be illustrated further in slides 20 – 23.



The BT Product

The statement BT ≥ 1, sometimes called the Uncertainty Principle, tells us that if we choose a very small bandwidth B, then we need a corresponding large measurement time T. This is based on the laws of physics and cannot be ignored! We must respect the Uncertainty Principle (also called the BT product) whenever we perform any kind of frequency analysis (sound, vibration, light, etc.)

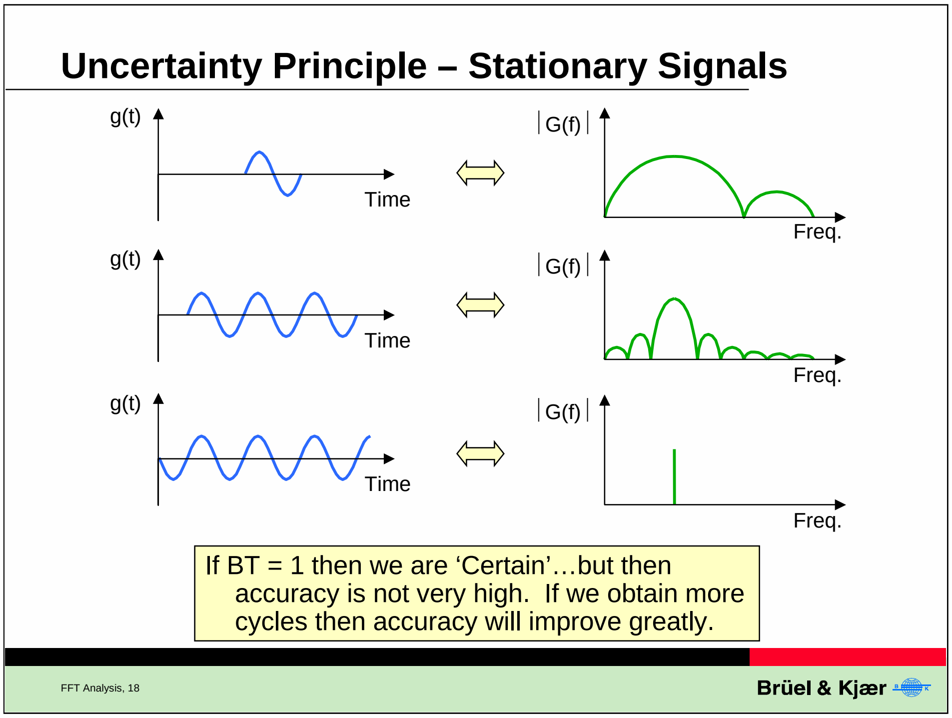
As an example: If we need a narrow band analysis with resolution better than 1 mHz, then we must wait at least 1 000 sec (~ 17 min) for the result to be valid. And the signal must be stationary within 1 mHz over this period of time as well, for it to make sense to choose such a high resolution.



Uncertainty Principle Example

Here we can see an example of how the Uncertainty Principle is applied. If you select an FFT with the above settings (Lines = 800, Span = 800 Hz), then you will get a df (resolution) of 1 Hz (remember that df, resolution, is calculated by dividing Span by Lines). Once this is done we can see that in order to satisfy the BT product (Uncertainty Principle), we must measure for at least 1 second.

Another way to better understand the Uncertainty Principle is to think of it this way: In order to be 'certain' that we have accurately measured a sine wave, we must measure at least one full cycle of that sine wave. If we measure less than one full cycle we cannot be 'certain' that we have actually observed the sine wave in question. As an example, in order to be certain that we have measured a 100 Hz sine wave, we must capture one full cycle of 100 Hz (which is 0.01 seconds…time = 1/bandwidth). If we only captured half of that sine wave, say 0.005 seconds, we cannot be 'certain' that the observed phenomenon is truly a 100 Hz sine wave.



Uncertainty Principle

Before going into a discussion of the Fourier Transformation, let’s have a look at the most important principle of frequency analysis.

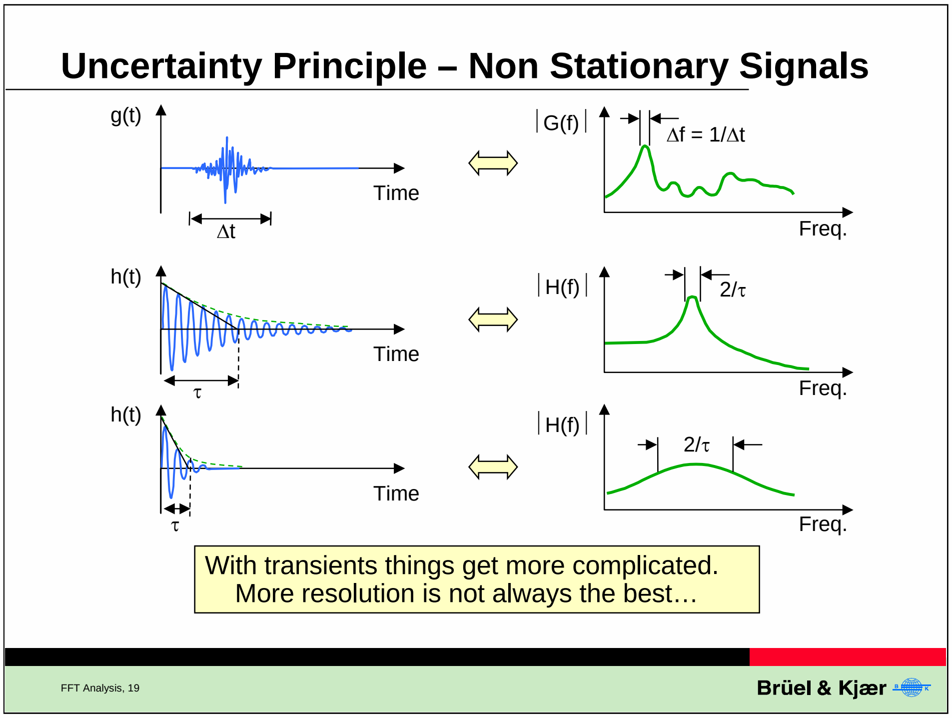
Imagine that we have a tone burst of a 1 kHz sine wave. If we measure over an infinite amount of time, years, millenia, we would be able to determine the frequency of that signal with enormous accuracy. But no one has time to measure for such a long period.

To go to the opposite extreme, imagine that you only picked out one period of the sine wave. If the frequency is 1 kHz it turns out that the resolution in the frequency domain, the “bandwidth” of that frequency spectrum, is also 1 kHz. The uncertainty is just as large as the frequency you want to identify.

If we use a little more time and extend the measurement period to include three periods we will see that the “bandwidth” of the analysis is reduced to 300 Hz.

If we extend the measurement time to cover 10 periods the “bandwidth” will be reduced to 100 Hz and so on…

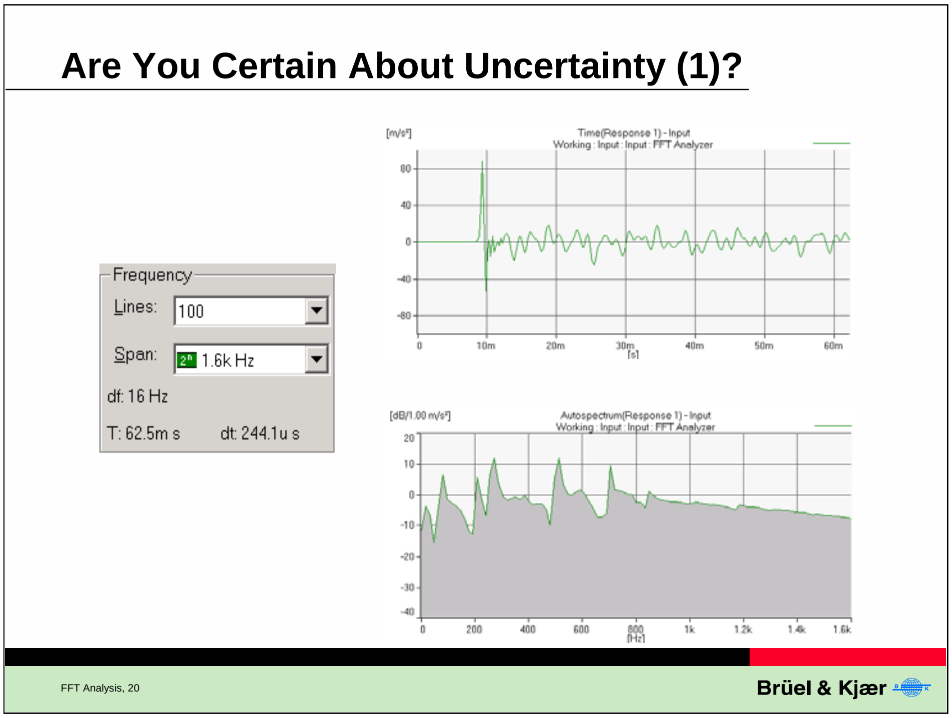
This can be simply summarised by the Uncertainty Principle: $\Delta t \cdot \Delta f \geq 1$, where Δt is the measurement and Δf is the filter “bandwidth” in the frequency domain. To reduce the bandwidth Δf you need to increase the measurement period Δt.



Uncertainty Principle

Another consequence of the uncertainty principle is that if we have a transient which only lasts for a time period of Δt, the bandwidth of the narrowest peak in the spectrum will be 1/ Δt.

If we look at the impulse response of a system, we find that the bandwidth of the peak in the frequency response is 2/τ, where τ is the decay time for the resonance. We will see later how this very important fact is used in connection with modal analysis to estimate the modal behaviour of structures.

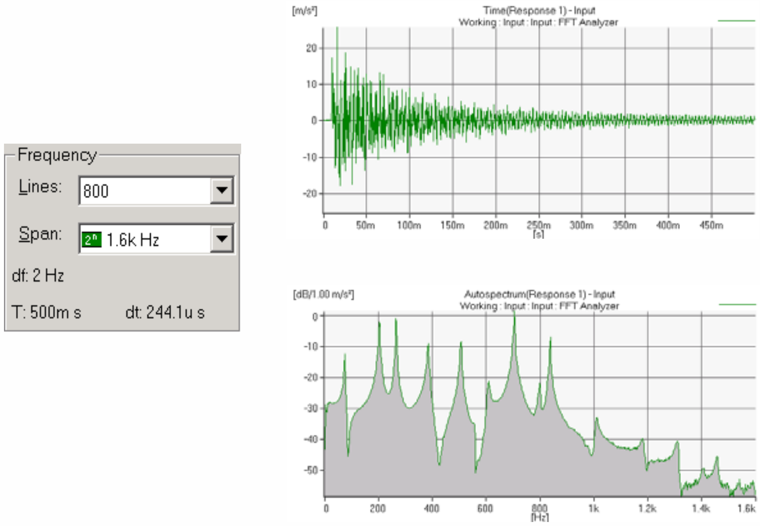


Are you Certain…(1)

Here is a real world example of what happens when trying to analyse a transient signal using an FFT analyzer. In the top there is a time block, whose length is determined by the T of the FFT analyzer, with a corresponding frequency spectrum. The FFT analyzer settings are shown on the left with all of the important information that was discussed earlier. For this test I simply took a hammer and struck a metal plate, then measured the resulting output using an accelerometer.

In this first slide I used an FFT with the settings of 100 Lines, 1.6kHz Span. Based on the math that we already know, this means that our T (time block) will be 62.5 ms long. At first glance, the FFT looks ok. We have some noticeable peaks and there does not appear to be a lot of noise interfering with the data. If no other measurements were made we may consider this to be a good result…

Are You Certain About Uncertainty (2)?



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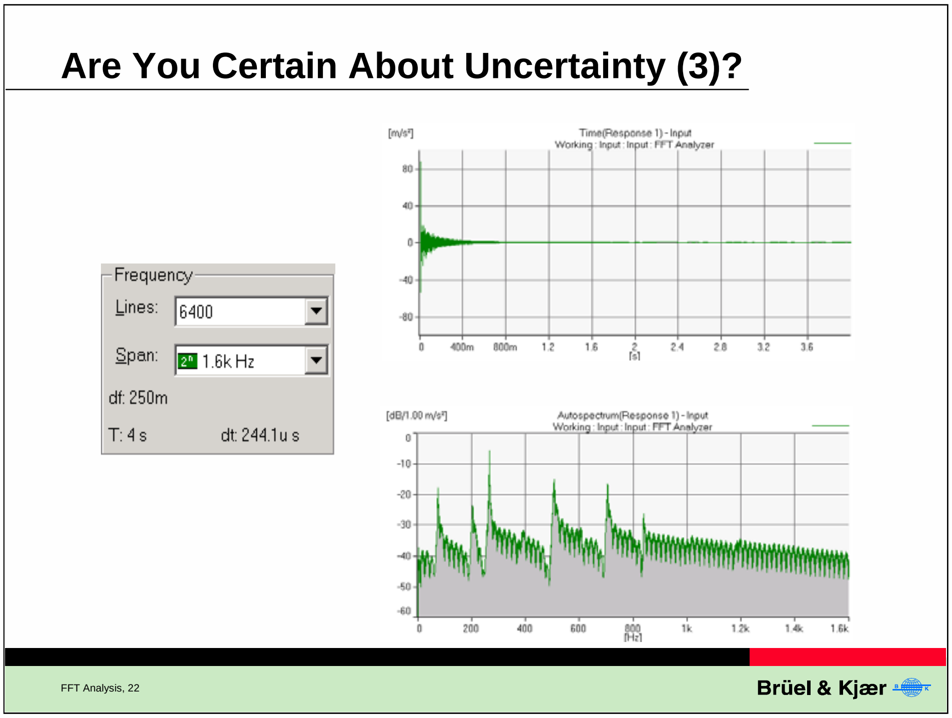
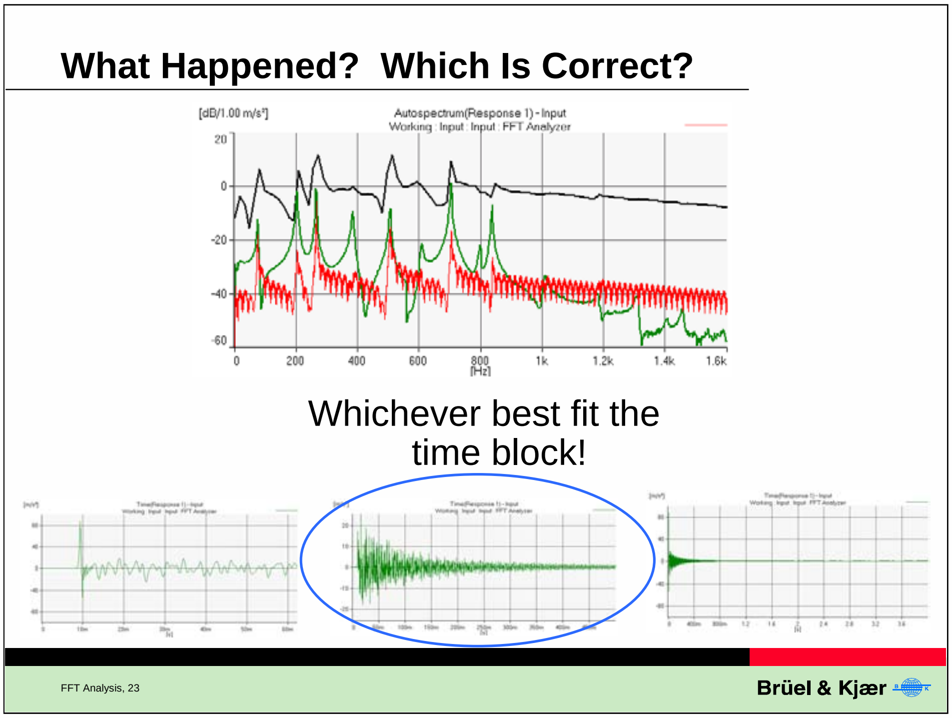
Are you Certain…(2)

Now let's up the resolution to 800 Lines, but keep the Span at 1.6kHz. This gives us a T (time block) of 500 ms and a df (resolution) of 2 Hz. Notice the significant difference in the time and frequency plots vs. the data on slide 20! More time data is gathered and the frequency spectrum is much more defined. There are even some peaks in this FFT that do not appear in the FFT on slide 20. Hmm, what happens when…



Are you Certain…(3)

Let's increase the Lines once more to 6400 Lines, and keep span the same at 1.6kHz. Again, notice the difference in the T (time block) and df (resolution). To achieve the higher resolution, 250mHz, we need to capture a very long time block, 4 seconds (per the Uncertainty Principle). Notice how much more time data we gather as compared to slides 20 and 21. Especially notice how much time data is at or near 0 m/s^2. All of this excess time with no meaningful amplitude data will essentially 'average' the amplitude down in the FFT. The peaks in the FFT are very narrowly defined, but there appears to be a lot of noise in the spectra.



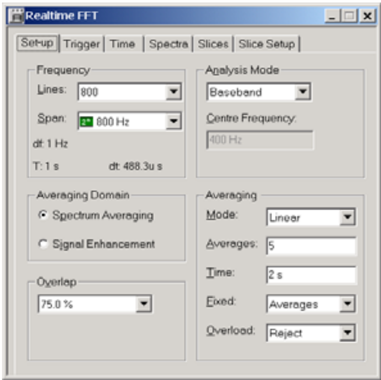
Are you Certain…(Conclusion)

If we overlay all of the FFT spectra on top of one another we can see that the amplitudes vary dramatically between the three measurements. If you look closely we can still pick out *most, but not all* of the frequencies. Keeping in mind that this is the same exact data measured using three different FFT resolutions. The obvious question becomes: which FFT is correct?

The answer: the correct FFT is the one that had the best time block fit. In this case, the FFT with 800 Lines (which yielded a T of 500 ms) best fit the data measured. Therefore, this FFT will give us the best measurement. This makes sense when all values are overlaid because you can see that the low resolution FFT (100 lines) in black does not pick out all of the frequencies and overestimates the amplitudes, and the high resolution FFT (6400 lines) in red has a lot of noise in the spectra and seems to skip a few frequencies. Why? The low resolution only used a short time block (62.5 ms), which probably did not capture all frequencies of interest (remember the Uncertainty Principle?). The high resolution used a very long time block, which then gathered a lot of dead time, averaging the amplitude values down.

The morale of this story: do not use super high resolution just because your analyzer can do it! Always think before hand about how much time your event will cover. If you are measuring a stationary signal, you can use a lot of resolution because the event will repeat many times in a long time block…but when analysing transient events, you must be conscious of the length of time that transient will occur and choose the best time block for that event RATHER THAN choosing a lot of resolution just because your analyzer can. This is common mistake people make when using an FFT analyzer so be sure to fully understand this concept!

Ensuring Repeatable Measurements



- Think about the signal you are measuring
  - Stationary
  - Transient
  - Combination…
- Always report:
  - Lines
  - Span
  - Window used
  - Overlap
  - Averaging Type
  - # of Averages
  - Start Trigger?

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Ensuring Repeatable Measurements

Whenever you use an FFT analyzer it is critical that you note the settings of that analyzer along with the actual data. If you do not note these settings, then it will be difficult to ensure that someone else (your customer, your vendor, your consultant, etc.) will measure the data and come up with the same results (remember slide 23?). In this slide, you can see a list of all necessary parameters that should be reported whenever measuring data with an FFT analyzer. If you list these items with your data, anyone else who makes a measurement with an FFT set up the same way will be more likely to come up with the same results. If not, look at slide 23 again!

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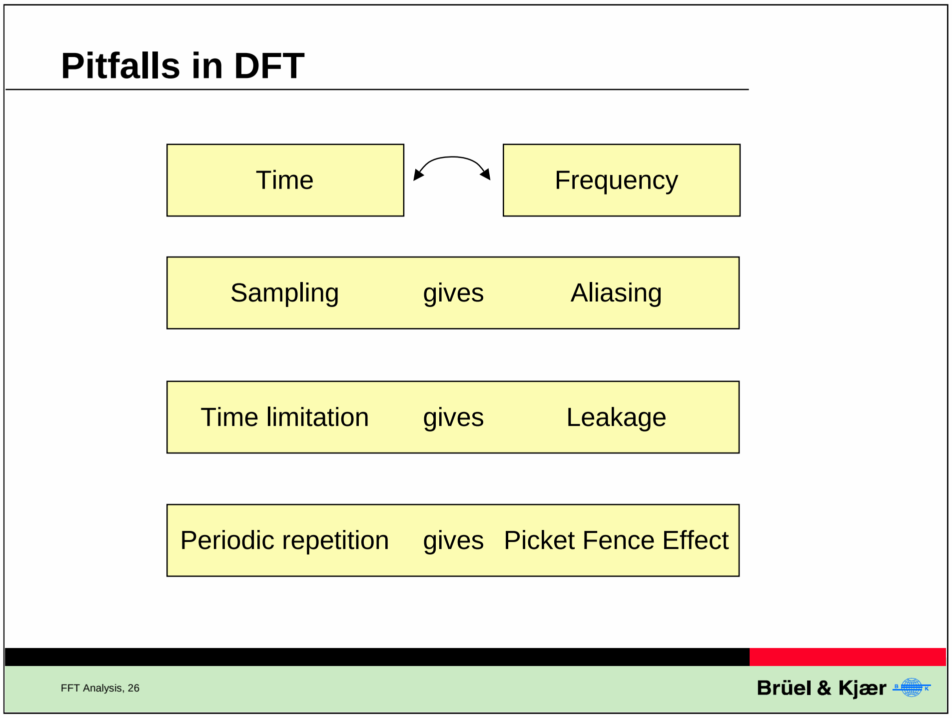


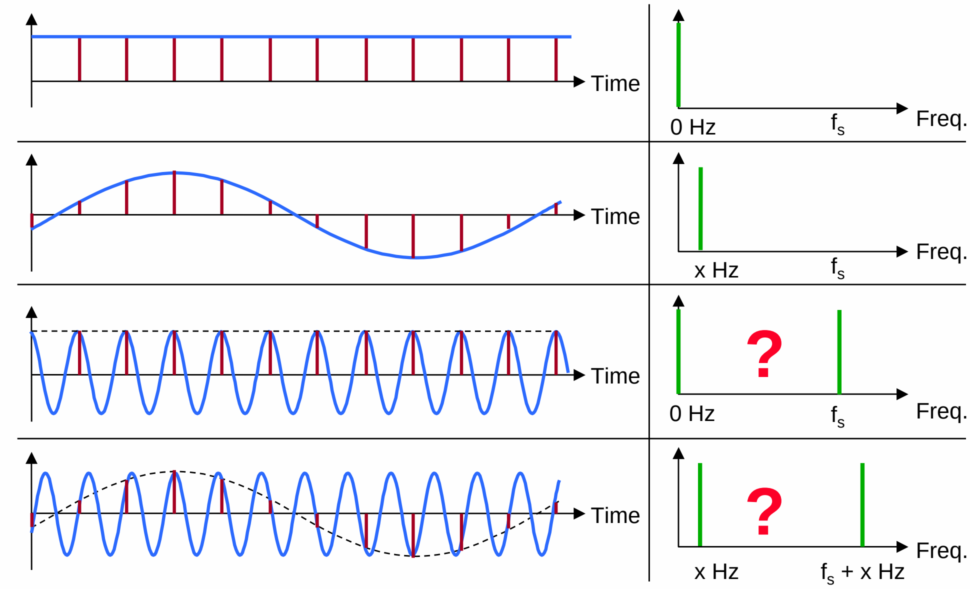
Pitfalls in Discrete Fourier Transform

Here the pitfalls of DFT analysis are summarised.

Sampling of the time signal results in aliasing in the spectrum. The time limitation results in leakage, and the periodic repetition of the time signal in picket fence effect.

Despite these pitfalls, FFT analysis is a very powerful analysis method and during this lecture we will look at means for minimising the errors resulting from these effects.

Aliasing



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Aliasing

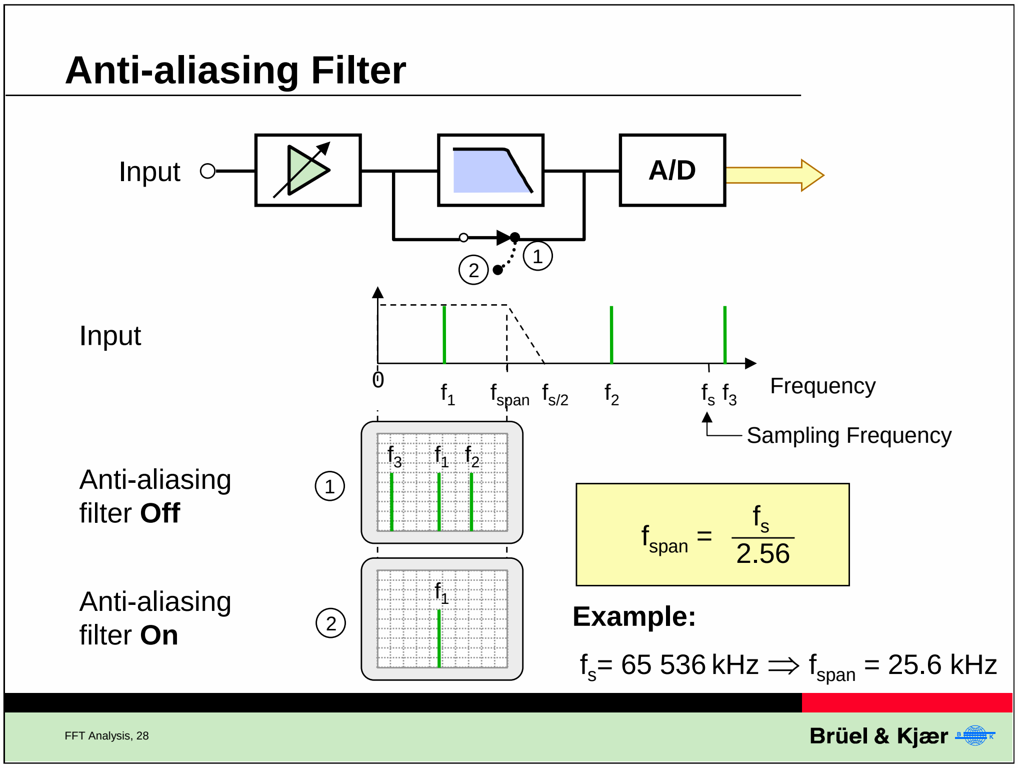
If we look at these four signals again and look at what happens when we sample the signals we will see that the samples in the two case at the bottom could represent two different frequencies.

In the third case we see that a frequency identical with the sampling frequency could just as well represent 0 Hz (= dc).

In the fourth case we see how the sampling of a signal with a frequency x Hz above the sampling frequency results in samples which just as well could represent x Hz. A signal x Hz below the sampling frequency would be represented by the same samples.

This is aliasing.

The way to overcome the problem is to remove all frequency components above half the sampling frequency (also called the Nyquist frequency) thereby removing the possible source of error. See next slide.

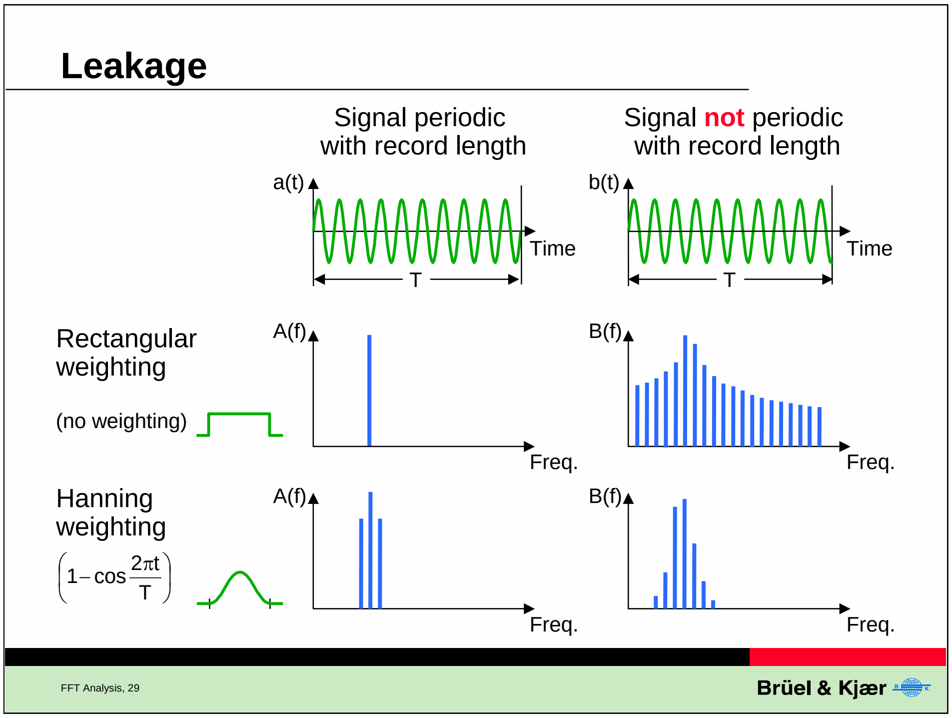


Antialiasing Filter

Aliasing has been discussed in a previous lecture but here is a summary.

By passing the signal to be analysed through an analogue low pass filter before it is sampled, it is possible to remove all frequency components above half the sampling frequency thereby removing the possible source of error.

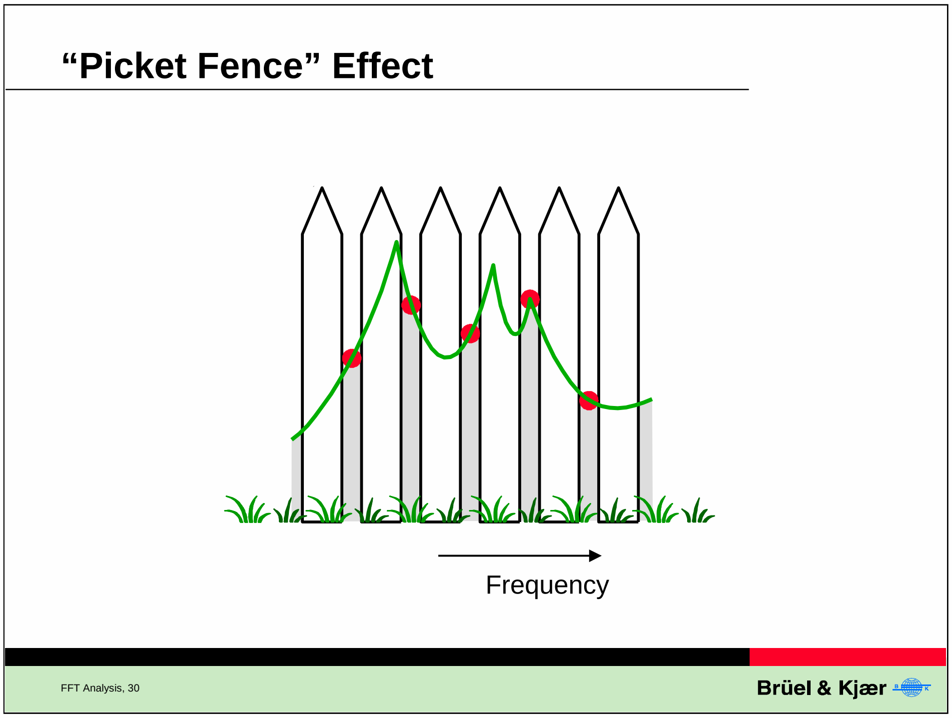
Note how the two frequencies above $f_s/2$ appear in the range 0 to f_{span} (maximum analysis frequency), when the input signal bypasses the antialiasing filter.



Leakage

Leakage is a problem which is a direct consequence of taking only a finite length of time history. The problem is illustrated here by two examples. Two sinusoidal time signals with a slightly different frequency are subjected to the same analysis process. In the first case, the signal is perfectly periodic with the record length T, and the resulting spectrum is a single line, at the frequency of the sine wave. In the second case, the signal is not periodic with the record length giving a frequency spectrum which appears much broader than what it should be. The reason is that energy has “leaked” into a number of the spectral lines close to the true frequency.

The way to overcome this problem is to apply a time weighting to the signal before analysis as illustrated at the bottom where the signal has been weighted by a Hanning weighting. Later in the lecture we will have a closer look at different weightings and discuss where to use them.



"Picket Fence" Effect

The fact that the spectrum has become discrete is called the "Picket Fence" Effect.

The reason for this is illustrated here. The fact that only discrete values of the spectrum are available corresponds to looking at the spectrum through a picket fence.

Picket fence error is typically an underestimation of amplitude values. Choice of an appropriate weighting function can minimise this error

How to Avoid the Pitfalls of DFT

1. Aliasing: Caused by sampling in time

   Solution:
   - Use anti-aliasing filter (f_c) and sampling rate f_s>2 f_c

2. Leakage: Caused by time limitation

   Solutions:
   - Use correct weighting (signals)
   - Increase the frequency resolution (systems)

3. Picket fence effect: Caused by sampling in frequency

   Solutions:
   - Use correct weighting (signals)
   - Increase the frequency resolution (systems)

Pitfalls of Discrete Fourier Transform

As we have seen, certain precautions must be taken in order to make a good FFT analysis. In order to avoid aliasing, appropriate antialiasing filters must be used.

Similarly leakage errors and picket fence effect errors should be minimized by choosing the most suitable weightings. The errors will also be reduced by selecting a higher frequency resolution, except in some situations for sine waves.

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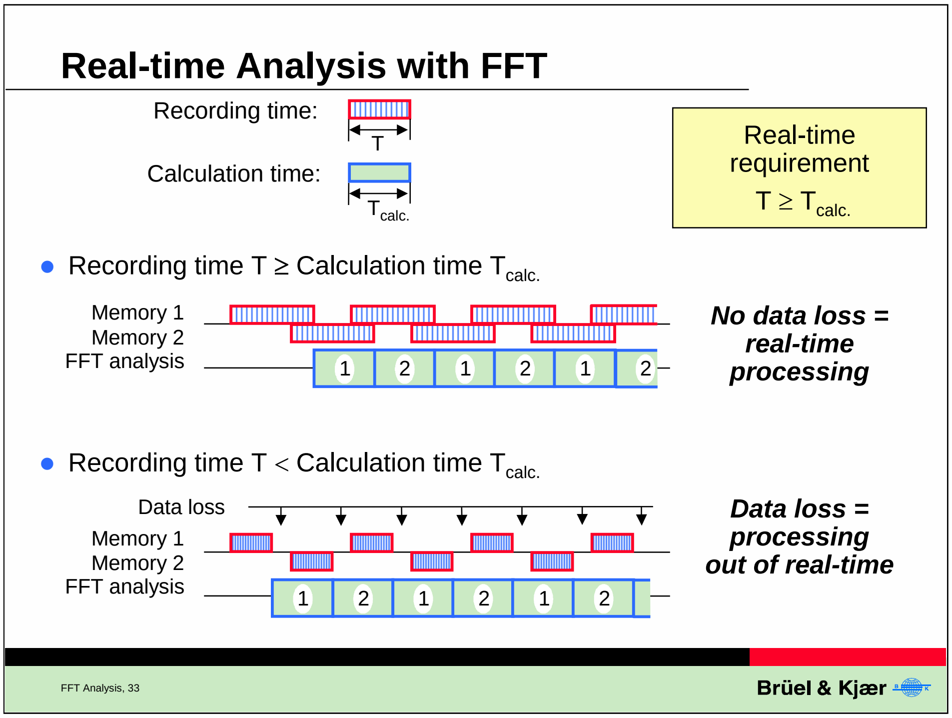
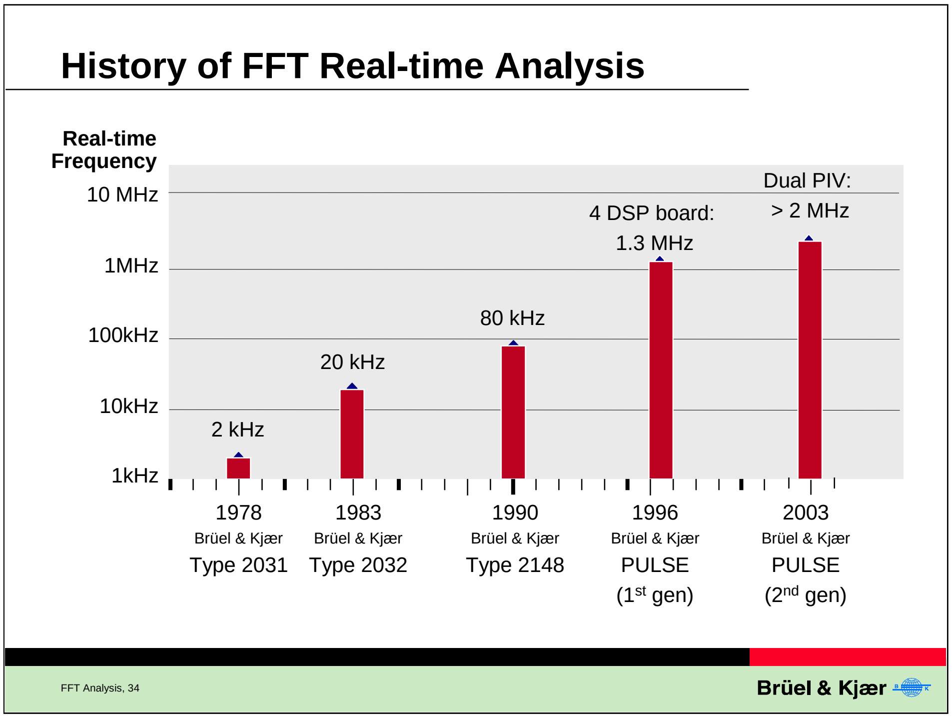


Real-time Analysis with FFT

An FFT analyzer does not always operate in real-time. This is a consequence of performing the analysis in blocks.

An FFT analyzer records a block of time data and while the spectrum of this time record is being calculated, a new time block is recorded in a parallel memory. If the analysis time - the time it takes to calculate and display a spectrum - is shorter than the record time, the analysis will be in real-time. But if the analysis time is longer than the record time, parts of the time signal will not be analysed and the analysis is no longer in real-time.

For a specific processing power, the analysis time depends on the transform size N, on the number of channels used in a multi-channel analyzer, on the applied weighting functions, and on the complexity of the function shown on the screen. The record time depends on the frequency range and on the number of frequency lines.



History of FFT Real-time Analysis

The real-time frequency of an FFT analyzer depends on the calculation speed of the hardware and with the increasing speed of microprocessors we have seen how the real-time frequency of analyzers has also increased tremendously over the last few years.

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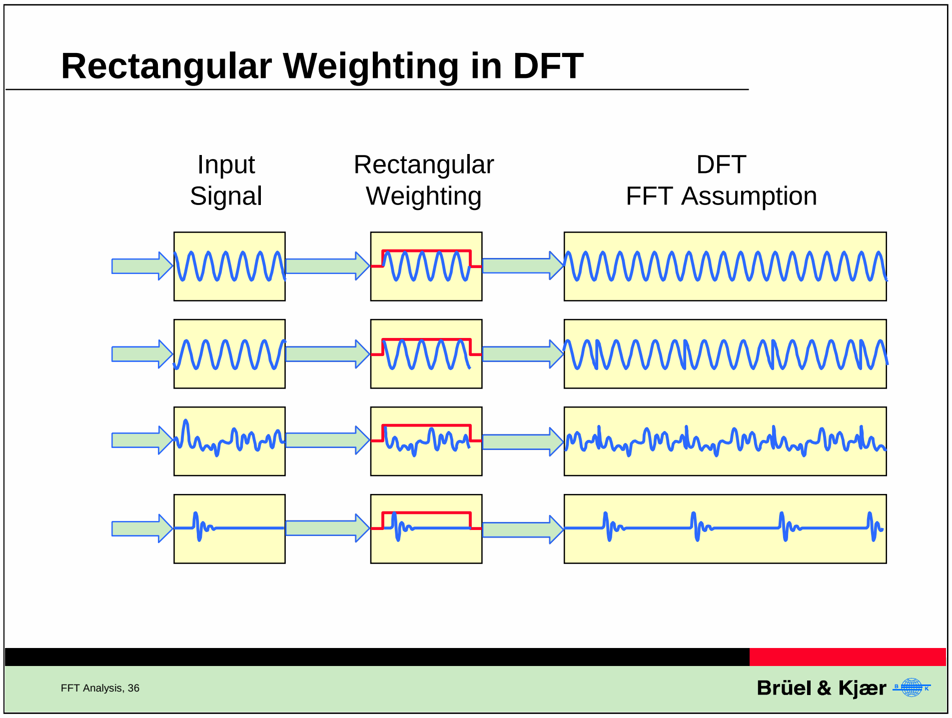
- Pitfalls of an FFT Analyser
- Real-time Analysis
- **Time Weighting**
- Overlap Analysis
- Signal Types and Spectrum Units
- FFT Summary



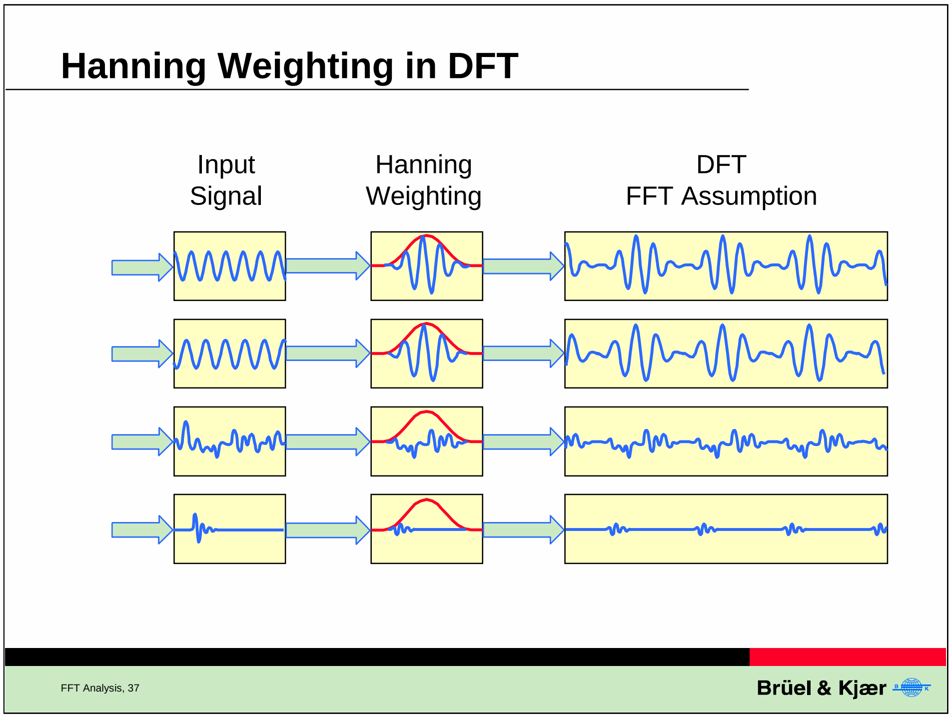
Rectangular Weighting in Discrete Fourier Transform

This illustrates how the Discrete Fourier Transform assumes that the analysis is performed on an infinite signal going from - ∞ to + ∞ whereas in reality is based on only one block of data. If we imagine this block of data as being repeated over and over again we see some very abrupt discontinuities in some situations.

In the first situation shown here, the record length equals an even number of half sine waves whereas the record length in the second situation is equal to an uneven number of half sine waves giving rise to the very abrupt discontinuities.

In the third situation where it is a random signal being analysed we also see the discontinuities.

In the fourth situation we see a transient which starts and ends within the time record and the weighting function will therefore not affect the analysis result.



Hanning Weighting in Discrete Fourier Transform

By using the Hanning weighting we here see how this removes the discontinuities but at the same time distorts the signal.

Hanning weighting has strong advantages in analysis of continuous signals. The distortion made by the Hanning weighting has less effect on the spectra than the possible discontinuities caused by Rectangular weighting.

For transients, however, Hanning weighting may distort the signals seriously while Rectangular weighting, as discussed earlier, causes no problems as long as the signal starts and stops within the record.

In the following we will look at how the different weightings affect the analysis result.

Use of Weighting Functions in *Signal* Analysis

	Weighting					
	Rect-angular	Hanning	Transient	Expo-nential	Kaiser-Bessel	Flat Top
Transients:						
• General purpose		✓ + overlap				
• Short transient	✓		✓			
• Long decaying transients				✓		
• Very long transients		✓ + overlap				
Continuous signals:						
• General purpose, RTA		✓				
• Two-tone separation					✓	
• Calibration						✓
• Pseudo random	✓					

Use of Weighting Functions in Signal Analysis

A summary of the weightings to be used for different kinds of signals is shown here.

For transients shorter than T, Rectangular weighting can always be used, but for very short transients the Transient weighting gives a better signal to noise ratio. Exponential weighting will reduce the problem of leakage due to truncation for decaying transients longer than T. For transients much longer than T, Hanning weighting can be used with an overlap of 66.6% for instance, giving a flat overall weighting. This will be discussed in detail later.

For continuous signals, the general purpose weighting is the Hanning weighting. It gives fairly good results and it is very fast. This is important when real-time analysis is required.

The Kaiser-Bessel weighting is used especially for separation of two closely spaced frequencies with large amplitude difference, since the selectivity is very good.

The Flat Top weighting is mainly used for calibration purposes since the amplitude error is negligible.

For pseudo random signals, Rectangular weighting should be used. Pseudo random noise consists of sine waves with frequencies coinciding with the analyzer lines. Hence there is no leakage using Rectangular weighting.

Rectangular weighting is sometimes preferred in order analysis where sampling of the signal is controlled by the source being investigated. This means that the fundamental frequency and the harmonics will always coincide with the analyzer lines. However, if the signal also contains components which are not related to the sampling frequency, Rectangular weighting will give too much smearing of these components, and Hanning weighting should be used.

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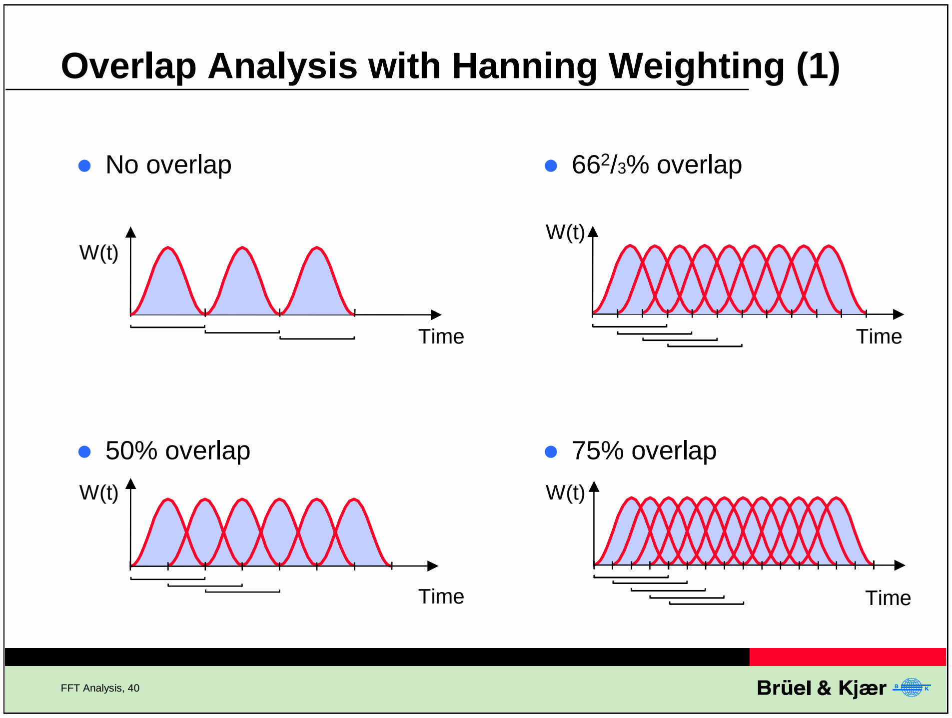


Overlap Analysis (1)

When using other weighting functions than the Rectangular weighting, the condition $T \geq T_{calc.}$ is not sufficient to avoid loss of data and thereby possible loss of valuable information. To illustrate this, Hanning weighting is used. If the time records do not overlap, parts of the signal will not be included in the average. To avoid this, overlap processing can be used.

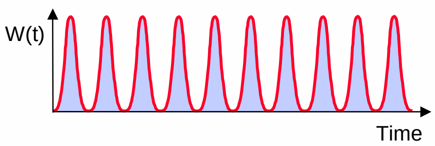
For Hanning weighting overlap of 50%, $66^2/_3$% or 75% overlap are often used. But this requires that the conditions $T \geq 2\ T_{calc.}$, $T \geq 3\ T_{calc.}$, or $T \geq 4\ T_{calc.}$ respectively are fulfilled.

For 50% overlap, two successive time records of random noise will be almost completely uncorrelated. This provides an extra advantage of overlap analysis: A certain accuracy in the estimate of a spectrum can be obtained in approximately half the time necessary without overlap. But again, this is only possible in a frequency span up to half the normal real-time bandwidth.

Overlap Analysis with Hanning Weighting (2)

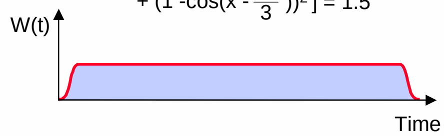
- No overlap

$(1 - \cos x)^2 = 1 - 2\cos x + \cos^2 x$



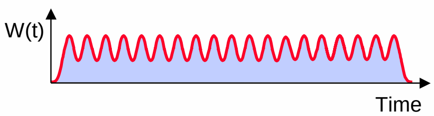
- $66^2/_3$% overlap

$1/3\ [\ (1 - \cos x)^2 + (1 - \cos(x - \frac{2\pi}{3}))^2 + (1 - \cos(x - \frac{4\pi}{3}))^2\ ] = 1.5$



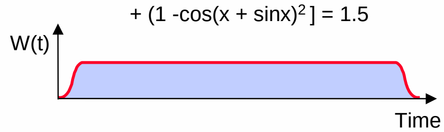
- 50% overlap

$1/2\ [(1 - \cos x)^2 = (1 + \cos x)^2\ ]\ 1 = \cos^2 x$



- 75% overlap

$1/4\ [\ (1 - \cos x)^2 + (1 - \sin x)^2 + \cos x)^2 + (1 - \cos(x + \sin x)^2\ ] = 1.5$



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Overlap Analysis with Hanning Weighting (2)

Here the overall weighting function for Hanning weighting with 0 %, 50 %, $66^2/_3$%, and 75 % overlap are shown.

It is seen that 66 2/3 % and 75 % weighting give equal weighting. 50 % does not give equal weighting since the weightings must be squared before they are added to give a measure of the power in the signal being analysed. Therefore the "real" real-time bandwidth for Hanning weighting is determined by the condition $T \geq 3\ T_{calc.}$ and not the condition $T \geq T_{calc.}$ as is the case for Rectangular weighting.

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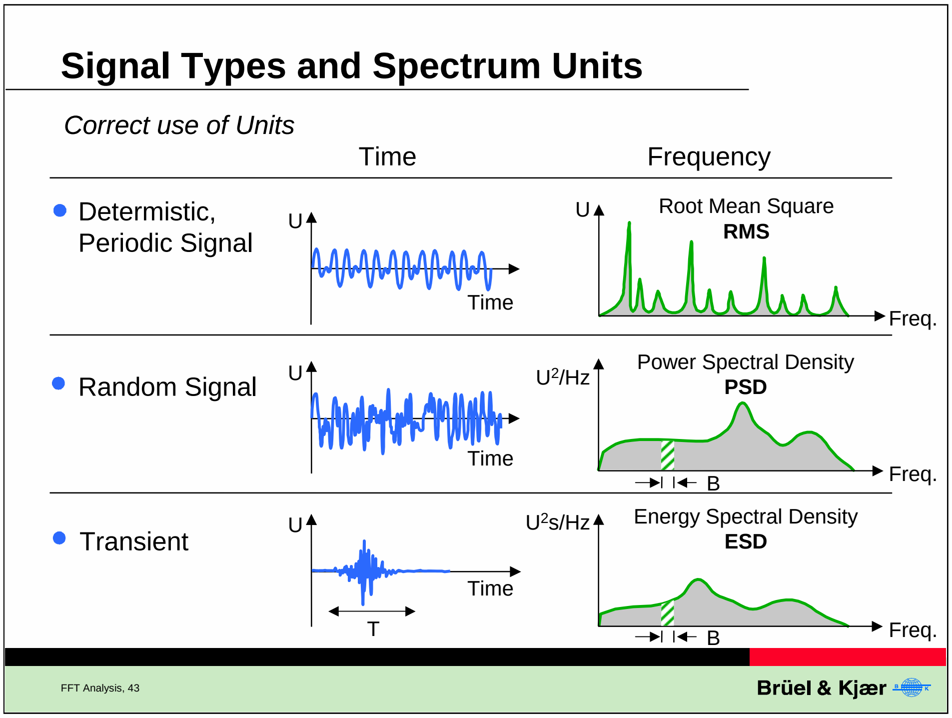

Correct use of Units

It is extremely important that the correct units are used for the different types of signals.

This is described in more detail in another lecture, so here we only include a summary.

FFT - Summary

The Discrete Fourier Transform:

- The DFT has properties very similar to the integral Fourier Transform
- The DFT has certain pitfalls: aliasing, leakage and picket fence effect
- Recording time, sampling interval, sampling frequency, frequency span and frequency resolution are all related

Weighting functions, leakage and picket fence effect:

- Many different weightings exist for different purposes
- Use of the proper weighting can reduce leakage and picket fence effect errors
- Weightings can be regarded as filters

Real-time Analysis, Overlap Analysis and Triggering:

- Condition for real-time analysis: $T \geq T_{calc.}$
- Other weightings than rectangular may require overlap analysis to avoid loss of data or to get a flat overall weighting function
- Many different trigger functions exist for different purposes

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FFT - Summary

A summary of the main conclusions of this section about FFT analysis is given in this figure.

Literature for Further Reading

- Frequency Analysis by R.B.Randall
  (Brüel & Kjær Theory and Application Handbook BT 0007-11)
- The Fast Fourier Transform by E. Oran Brigham
  (Prentice-Hall, Inc. Englewood Cliffs, New Jersey)
- The Discrete Fourier Transform and FFT Analyzers by N. Thrane
  (Brüel & Kjær Technical Review No. 1, 1979)
- Zoom-FFT by N. Thrane
  (Brüel & Kjær Technical Review No. 2, 1980)
- Dual Channel FFT Analysis by H. Herlufsen
  (Brüel & Kjær Technical Review No. 1 & 2, 1984)
- Windows to FFT Analysis by S. Gade, H. Herlufsen
  (Brüel & Kjær Technical Review No. 3 & 4, 1987)
- Who is Fourier?
  (Transnational College of LEX, April 1995)

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